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Software note

EWSmethods: an R package to forecast tipping points at the community level using early warning signals, resilience measures, and machine learning models

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Early warning signals (EWSs) represent a potentially universal tool for identifying whether a system is approaching a tipping point, and have been applied in fields including ecology, epidemiology, economics, and physics. This potential universality has led to the development of a suite of computational approaches aimed at improving the reliability of these methods. Classic methods based on univariate data have a long history of use, but recent theoretical advances have expanded EWSs to multivariate datasets, particularly relevant given advancements in remote sensing. More recently, novel machine learning approaches have been developed but have not been made accessible in the R (www.r-project.org) environment. Here, we present *EWSmethods* – an R package (www.r-project.org) that provides a unified syntax and interpretation of the most popular and cutting edge EWSs methods applicable to both univariate and multivariate time series. *EWSmethods* provides two primary functions for univariate and multivariate systems respectively, with two forms of calculation available for each: classical rolling window time series analysis, and the more robust expanding window. It also provides an interface to the Python machine learning model EWSNet which predicts the probability of a sudden tipping point or a smooth transition, the first of its form available to R (www.r-project.org) users. This note details the rationale for this open-source package and delivers an introduction to its functionality for assessing resilience. We have also provided vignettes and an external website to act as further tutorials and FAQs.

Keywords: bifurcation, critical, ecosystem management, ecosystem, resilience, time series, transition

Background

Natural systems are inherently non-linear and consequently challenging to forecast (Bradley and Kantz 2015, Pace et al. 2015). This has led to the application of dynamic system theory which aims to provide model-free and generic tools to identify approaching non-linearity (Scheffer et al. 2009, Clements and Ozgul 2018, Drake et al. 2019). The majority of this work has attempted to detect critical slowing down (CSD), a phenomenon displayed by systems as they approach a bifurcation or 'tipping point' (Kuehn 2011, Strogatz 2015). In brief, CSD manifests when, as the distance to the tipping point decreases, the ability of the system to recover from perturbations and return to its average trend also decreases (Wissel 1984). This stems from the dominant eigenvalue of the system trending towards zero and results in successive snapshots in time being more similar than if the system was far from a tipping point; in practical terms, successive abundance/biomass measurements in time or space begin to correlate more strongly.

Detecting CSD can be as simple as tracking the temporal change in summary statistics. For example, increasing autocorrelation at lag-1 (Dakos et al. 2010), increasing variance (Carpenter and Brock 2006), increasing skewness (Guttal and Jayaprakash 2008) and kurtosis (Biggs et al. 2009) are all representative of CSD. In univariate data, each of these have been successful in identifying oncoming tipping points in simulated experiments (Dakos et al. 2012b, Kéfi et al. 2013) as well as empirical lake regime shifts (Biggs et al. 2009, Carpenter et al. 2011), boreal forest loss (Rogers et al. 2018), disease (re)emergence (Harris et al. 2020, O'Brien and Clements 2022), and psychopathology (McSharry et al. 2003, Schreuder et al. 2020). The popularity and scope of EWSs is consequently expanding to new applications (Cailleret et al. 2019) and multivariate data sources (Weinans et al. 2021, Baruah et al. 2022) to maximise the utility of the approach with the increasingly large amounts of ecological monitoring data now available (Meinson et al. 2015, Jucker et al. 2017, Besson et al. 2022). There is therefore a general desire to exploit EWSs in both traditional research and policy decision making as evidenced by the rapid increase in the publication and citation of EWS literature per year.

Multivariate forms of EWSs (Weinans et al. 2019, Lever et al. 2020, Medeiros et al. 2022) and deep learning models (Bury et al. 2021, Deb et al. 2022) are of particular interest as they appear superior tools to the univariate signals described above. Multivariate approaches exploit information from multiple measurements of a shared system (e.g. multiple species in an ecosystem or multiple sensors in a combustion engine) to provide an overall signal of system resilience. Pooling information in this way buffers against the uncertainty of choosing which data source should be assessed. For example, the trophic level of EWS assessment influences the strength of signal observed in simulated communities (Patterson et al. 2021), and whilst the authors provide guidance on the optimum species/time series to monitor, the required information to identify those time series may not be available to empirical users. Multivariate EWSs can therefore provide a naïve yet robust assessment for multivariate data in the absence of complete information.

Similarly, CSD may not be the only signature of systems close to a tipping point. Our identification of the phenomenon stems from linear stability analysis (LSA) of mathematical models (Ludwig et al. 1978, Scheffer et al. 2009), but machine learning tools can identify other phenomenological features not detected from LSA. For example, machine learning models trained upon transitioning data outperform equivalent models trained upon the EWSs of the same transitioning data (Deb et al. 2022). This is indicative of alternative features being more informative than CSD to warrant general usage, although the 'black-box' nature of the approach limits its accountability (Enni and Herrie 2021). This being said, multiple machine learning models are now available for transitioning systems that improve the transparency of predictions by training on simple mathematical models associated with LSA (Bury et al. 2021, Deb et al. 2022). These models can consequently build upon our foundational knowledge of tipping points by taking advantage of the biases inherent in their training.

Currently, neither multivariate nor machine learning approaches have functionality for R (www.r-project.org) users and resultingly there is a need for simple tools to interact with the variety of EWS approaches available to researchers. Certain EWS functionality has previously been provided by the *earlywarnings* R package (www.r-project.org, Dakos et al. 2012a), however the package is limited to one form of EWS calculation (rolling windows) in univariate data only. There have also been advances using alternative methodologies such as expanding window and composite EWSs, which introduce data in an add-one-in fashion to provide a standardised time series of EWS strength (Drake and Griffen 2010, Clements and Ozgul 2016). This second approach improves the reliability of EWS predictions in univariate data (Southall et al. 2022) but is not currently available in an easy-to-use form. Unfortunately, many of the custom functions written to facilitate this research are limited to the subscription MATLAB product (MathWorks 2022) or hidden in publications' supplementary information (e.g. composite EWSs – Clements et al. 2019, O'Brien and Clements 2022). In combination, this has limited the accessibility of EWS development to the wider community.

Compiling these various functions in to a single and comprehensive R package (www.r-project.org) whilst rectifying computational errors is required to increase reproducibility of empirical ecological tipping point research and improve the interpretation and visualisation of results. We therefore designed the *EWSmethods* R package (www.r-project.org) to provide a suite of 'user-friendly' functions to predict critical transitions across both univariate and multivariate data sources and provide interpretable graphics. For univariate data, such as local fisheries or country level disease cases, EWSs can be estimated using either the rolling window approach of *earlywarnings* or the expanding window approach of Drake and Griffen (2010). The package also provides the user the capability to query the Python based EWSNet deep learning model

(Deb et al. 2022) in the R (www.r-project.org) environment and generate predictions on the time series' future. Thirdly, if multiple measurements have been made of a single system – such as when monitoring multiple species in the same community – multivariate EWSs can be estimated using either rolling or expanding window approaches. And finally, EWS distinct resilience metrics can also be estimated for univariate and multivariate data following the work of Ushio et al (2018) and Grziwotz et al (2023). *EWSmethods* therefore represents a compilation of new and existing tools to support this expanding field in an easy to use and interpret form. A comparison of the features *EWSmethods* provides vs the currently available *earlywarnings* package is provided in Table 1.

In this paper, we first describe the theory underpinning the methods used and the features of the *EWSmethods* package. We then highlight the practical use of the three modules to predict forthcoming transitions using a simulated multispecies dataset.

Methods and features

Time series data is the foundation of system monitoring and forecasting, leading to a massive diversity of time series forecasting methods and models developed to analyse them (De Gooijer and Hyndman 2006). CSD based indicators (i.e. the early warning signals (EWSs)) are no exception but require less technical expertise than traditional forecasting techniques (Dakos et al. 2015). This simplicity in calculation holds for both univariate and multivariate assessments.

Univariate early warning signals

EWSs developed for univariate data are the simplest form of CSD assessment and thus have received the most research effort. Table 2 describes the most common EWSs, all of which are provided in *EWSmethods* via the **uniEWS** function, and how they are calculated. Each of these are also provided in the *earlywarnings* package and mathematically described in detail (Dakos et al. 2012a). The development that *EWSmethods* provides over that package is the diversity of approaches used to compute these EWSs beyond those available in *earlywarnings*, allowing users to tailor their analyses to support their use case. This primarily involves the choice of rolling versus expanding windows during calculation (Fig. 1).

Rolling windows

The rolling window approach partitions the univariate time series of interest into a window of data points within which each indicator is estimated. The window then 'rolls' along the time series one data point at a time to update the indicator estimate and generate a new time series of EWSs (Fig. 1a). From this EWS time series, the Kendall's tau correlation of the EWS against time is used to generate 'warnings' (Fig. 1b). Specifically, if a strong tau correlation is found, this indicates an oncoming transition. The **uniEWS** function allows the user to specify the window size as a percentage of the time series' length and returns both the time series of EWSs and the estimated Kendall's tau to be interpreted.

Expanding windows

The alternative to the above computation differs by assessing change in an expanding window via a composite metric consisting of multiple indicators (Fig. 1c). The same EWS indicators as above are available to the expanding window approach (Table 2), but each indicator is standardised by subtracting its expanding mean from its calculated value at time t. This value is then normalised by division by its expanding standard deviation (Drake and Griffin 2010) – at each time point, the prediction is updated (Fig. 1d). A composite metric can then be constructed by summing all individual indicator values calculated per t. The resulting indicator value or score is hereafter referred to as 'strength'. If the indicator strength exceeds a threshold value, then a 'signal' has been identified. Typically, this threshold value is 2σ which is approximately equivalent to a 95% confidence interval and performs favourably compared to other threshold levels (Clements and Ozgul 2016, Clements et al. 2017).

The expanding window approach also allows multiple information sources to contribute to the assessment. For example, including body size estimates improves assessment reliability by reducing false positive rate whilst increasing the number of true positives (Clements and Ozgul 2016, Baruah et al. 2020). **uniEWS** consequently accepts a trait argument where

Table 1. Comparison of supported features between the *EWSmethods* and *earlywarnings* R packages (www.r-project.org).

Feature	*earlywarnings*	*EWSmethods*
Rolling window early warning signals – univariate time series	✓	✓
Expanding window early warning signals – univariate time series	✗	✓
Rolling window early warning signals – multivariate time series	✗	✓
Expanding window early warning signals – multivariate time series	✗	✓
Machine learning model (EWSNet) – univariate time series	✗	✓
Maximum likelihood model-based approaches – univariate time series	✓	✗
Detrended frequency analysis and potentials – univariate time series	✓	✗
Sensitivity analysis – univariate time series	✓	✗
Fisher information, Jacobian estimates, etc – univariate and multivariate time series	✗	✓
Time series detrending	✓	✓
Time series deseasoning	✗	✓

Table 2. Description of the univariate early warning signal indicators provided by *EWSmethods* and their origin in the literature.

Indicator	Description	Reference
SD (Standard Deviation)	Increasing variance/standard deviation is observed approaching a transition, driven by critical slowing down (CSD)	Carpenter and Brock (2006)
CV (Coefficient of Variation)	Equivalent to SD as is simply SD at time t divided by the mean SD of the time series	Carpenter and Brock (2006)
AR1 (Autocorrelation at lag1)	Autocorrelation (similarity between successive observations) increases approaching a transition, due to CSD. The value of this indicator can be estimated as either the autocorrelation coefficient estimated from a first order autoregressive model or the estimated autocorrelation function at lag1	Held and Kleinen (2004)
Skewness	At a transition, the distribution of values in the time series can become asymmetric. This is skewness and can increase/decrease depending on the size of the alternative state	Guttal and Jayaprakash (2008)
Kurtosis	Kurtosis represents the system reaching more extreme values in the presence of a transition. Due to the increased presence of rare values in the time series, the tails of the observation distribution widen	Biggs et al. (2009)
Return rate	The inverse of the first-order term of a fitted autoregressive AR(1) model. Return rate is the primary quantity impacted by CSD – return rate decreases as a tipping point is approached	Carpenter et al. (2011)
Density ratio	Spectral reddening (high variance at low frequencies) occurs near transition. The density ratio quantifies the degree of reddening as the ratio of the spectral density at low frequency to the spectral density at high frequency	Kleinen et al. (2003)

an additional trait time series can be combined with the other 'abundance-based' EWSs as a composite metric.

Furthermore, the EWSs assessed using the expanding window approach can be improved using a consecutive signal strategy (Clements et al. 2019, Southall et al. 2022) where a 'warning' is only acknowledged when two or more signals are identified in a row. Southall and colleagues (2022) have recently showed that using this approach results in earlier and more reliable warnings over the rolling window approach.

Multivariate early warning signals

The second module contained in *EWSmethods* is the expansion of EWSs to multivariate data. The benefit of using multivariate techniques over univariate is that assessments of stability and proximity to tipping points can be performed at the system/community level rather than being constrained to the population level. Many of these multivariate EWSs have been tested and supported by Weinans et al. (2021)

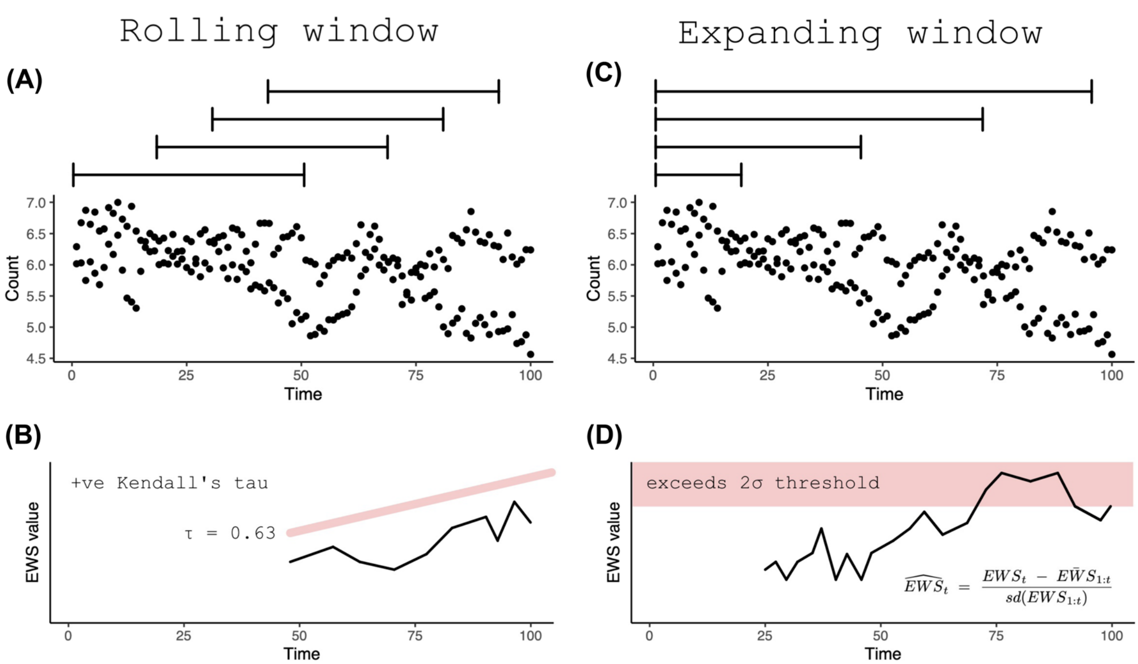

Figure 1. Visual representation of the difference between rolling and expanding window approaches to calculating early warning signals (EWSs – A vs C) in a hypothetical transitioning time series. Solid bars indicate the changing window. Panels B and D then indicate the quantity that represents a 'warning'. For rolling windows (A, B), this warning is a strong Kendall's tau correlation of EWS indicator values with time. Whereas, for expanding windows (C, D) a warning occurs when the standardised EWS value exceeds a 2σ threshold.

but open-source tools to calculate them remain unavailable. *EWSmethods* consequently provides multivariate EWS calculation via the **multiEWS** function.

There are two forms of EWS indicators appropriate for multivariate data: those averaged across all time series representing the system of interest (Dakos 2018), and those calculated from a dimension reduction (Held and Kleinen 2004, Weinans et al. 2019). The former is a simple technique to implement using just **uniEWS** but can be influenced by outlier time series, whereas the latter can display informative properties not identifiable in individual time series (Weinans et al. 2021). Unfortunately, their theoretical relationship with CSD is less well understood. *EWSmethods* and the **multiEWS** function therefore provides 12 multivariate indicators across both averaging and dimension reduction forms, each of which is described in Table 3.

Parameterisation of **multiEWS** is identical to **uniEWS** apart from the lack of capability for composite EWSs. This is due to it being currently unknown how combining multivariate EWS indicators influences their prediction reliability. Rolling and expanding windows are still available for multivariate EWSs and their interpretation remains the same as their univariate equivalents.

Machine learning model – EWSNet

The third *EWSmethods* module is an interface to the Python based EWSNet, a deep learning modelling framework for predicting critical transitions and tipping points (Deb et al. 2022). EWSNet consists of coupled long short-term memory and fully convolutional network sub-module routines, which together extract complex non-linear patterns from inputted time series to provide forecasts on the likelihood of oncoming tipping points. Details on the precise formulation and model structure can be found at Deb et al. (2022) and https://ews-net.github.io, whereas here we will focus on the application of EWSNet for ecologists and the setup of the R (www.r-project.org) environment to cooperate with EWSNet's Python backend.

The rationale behind EWSNet stems from the rapid success and widespread adoption of machine learning algorithms and their ability for learning patterns from data (Humphries et al. 2018). EWSNet exploits this ability by training models upon the simple non-linear mathematical models pioneered by ecological dynamic system research (Ludwig et al. 1978, Fraedrich 1978, Cheng et al. 2008, Scheffer et al. 2012, Kéfi et al. 2013). Specifically, these models encompass four forms of transition/tipping point - saddle-node (fold), pitchfork, supercritical Hopf, transcritical (Fig. 2a) – and include non-transitions to allow EWSNet to identify periods of stability. This combination of training results in three possible EWSNet predictions: critical transition, smooth transition or no transition. To aid interpretation of these predictions in real world systems, we suggest that a critical transition indicates oncoming sudden non-linearity, a smooth transition indicates a directional change in trend, and no transition indicates stability as outlined in Fig. 2b.

Table 3. Description of the multivariate early warning signal indicators provided by *EWSmethods*, their origin in the literature and which signal category they belong to.

Indicator	Description	Reference	Averaging or dimension reduction technique
Mean SD (Standard Deviation)	Average variance across all time series representing the system	Dakos (2018)	Average
Max SD	The variance of the time series with the highest variance of all assessed time series	Dakos (2018)	Average
Mean AR1 (Autocorrelation at lag1)	Average autocorrelation across all time series representing the system	Dakos (2018)	Average
Max AR1	The autocorrelation of the time series with the highest autocorrelation of all assessed time series	Dakos (2018)	Average
Dominant MAF (maximum autocorrelation factor) eigenvalue	The minimum eigenvalue of the system following MAF dimension reduction	Weinans et al. (2019)	Dimension reduction
MAF AR1	The autocorrelation of the data projected on to the first MAF – i.e. the autocorrelation of the first MAF	Weinans et al. (2019)	Dimension reduction
MAF SD	The variance of the data projected on to the first MAF – i.e. the variance of the first MAF	Weinans et al. (2019)	Dimension reduction
First PC (principal component) AR1	The autocorrelation of the data projected on to the first PC – i.e. the autocorrelation of the first PC	Held and Kleinen (2004)	Dimension reduction
First PC SD/ Explained variance	The variance of the data projected on to the first PC – i.e. the variance of the first PC	Held and Kleinen (2004)	Dimension reduction
Dominant eigenvalue of the covariance matrix	The maximum eigenvalue of the covariance matrix between all representative time series	Chen et al. (2019)	Neither
Maximum covariance	The maximum value of the covariance matrix between all representative time series	Suweis and D'Odorico (2014)	Neither
Mutual information	A measurement of multi-information or how much each time series informs on the others	Quax et al. (2013)	Neither

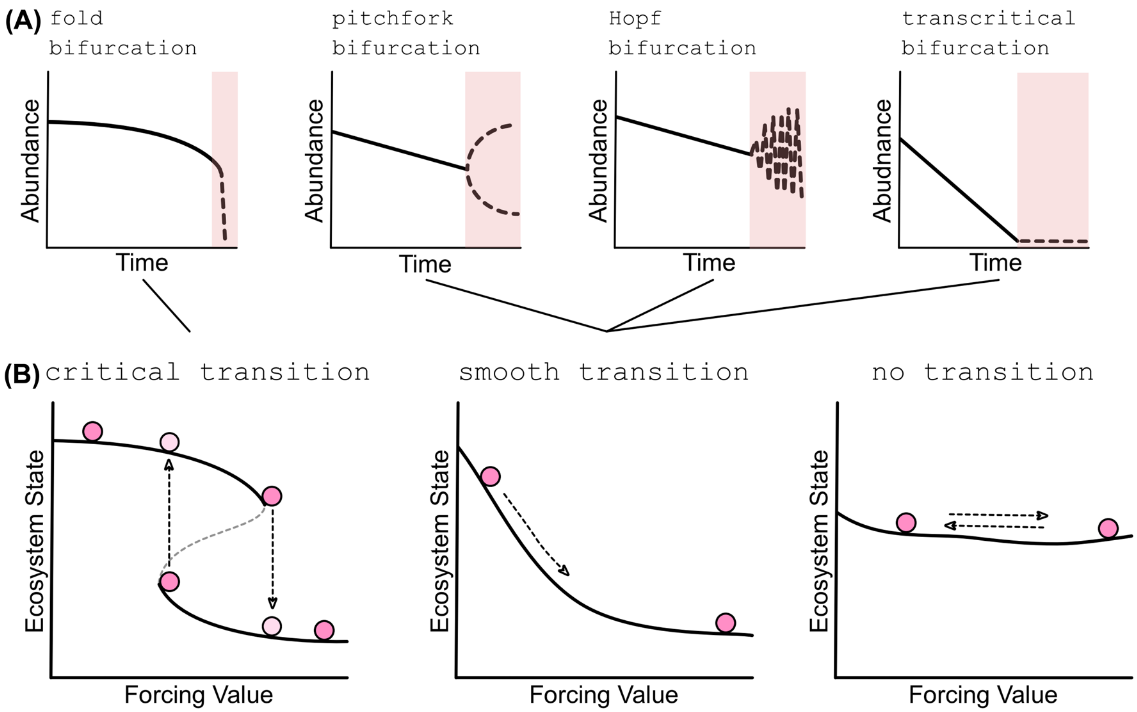

Figure 2. Visual representation of the four models EWSNet was trained (A) and their associated outcome in empirical time series (B). In panel A, the shaded region represents the period of transition with hatched lines indicate the new system trajectory. In panel B balls represent the position of the system of interest in a one dimensional stability landscape.

With machine learning tools limited for R (www.r-project.org) users, and EWSNet written in the Python language, the *reticulate* R package (www.r-project.org, Ushey et al. 2022) allows *EWSmethods* to call the Python functions required to load EWSNet and make predictions from user data. *EWSmethods* prepares the user's R (www.r-project.org) session to perform this interfacing via the **ewsnet_init** function. **ewsnet_init** loads a previously created Python environment with the Python packages required by EWSNet, or installs Python and initialises a new environment if either Python or the environment is not found. Due to the large file sizes being downloaded at this stage, **ewsnet_init** is verbose by default and requires user input to confirm that Python, the required packages, and environment should be downloaded and/or installed.

Users can then use **ewsnet_predict** to generate EWSNet predictions on a time series of interest. To date, EWSNet only supports only univariate time series, however the multivariate form of EWSNet is under active development. The current version of EWSNet also differs to that of the original authors by being robust to time series of variable length. This involved retraining using randomly sampled subsets of the data, ranging in length from 15 to 400 data points to better support the shorter time series available to empirical ecologists. Similarly, due to the variable magnitudes of ecological measurements, two sets of EWSNet's training weights are provided in *EWSmethods*, scaled vs unscaled (**ewsnet_reset** is required to download them); scaled models rescale the input data into the range 1–2. We recommend using scaled weights as they result in more conservative model predictions (O'Brien et al. 2023). **ewsnet_predict** then returns a prediction probability for each of the three potential outcomes ranging from 0.0–1.0. As EWSNet was trained on three possible outcomes, a probability of ~ 0.33 indicates all prediction outcomes are equally likely (1.0 divided by 3 equals ~ 0.33). Therefore, its authors suggest any probability greater than 0.33 implies a stronger than chance prediction and anything greater than 0.6 warrants serious scrutiny (Deb et al. 2022).

To summarise, EWSNet characterises the current dynamics of the observed time series relative to the various transition types on which it has been trained. From this training data, the model interprets the probability of the time series' trajectory belonging to each transition type. Estimated probability is therefore not the probability of transition in the future, but the probability that the time series shares characteristics of a transitioning time series.

Interpretation

EWSs are potentially powerful tools for managers. However, their interpretation can be complex and requires nuance. This is particularly true for rolling window approaches and EWSNet as it remains unclear what constitutes a 'strong' correlation or prediction probability. We however believe there are three approaches to defining an appropriate warning using EWSs. Firstly, a user may refer to a reference period for a baseline correlation, or track change in the strength of a signal through time (as in the expanding window approach above), where deviations from the general trend are informative. The second requires the user to define how conservative an assessment they require. For example, if the negative consequence of a transition is significantly larger than the consequence of acting upon a false positive, then a lower confidence warning may be appropriate (i.e. a low Kendall's tau coefficient/EWSNet prediction probability). And finally, the third requires comparing the observed signal to a distribution of signals generated via permutation of the original time series. If the observed signal is in the top x-th quantile of the distribution (the 95th quantile is commonly used) then a warning may be identified. The *EWSmethods* function **perm_rollEWS** provides this functionality, allowing the user to generate surrogate time series using three alternative permutation techniques: random sampling without replacement from the original time series (Theiler et al. 1992), simulation of an ARIMA model best fitting the original time series (Theiler et al. 1992, Dakos et al. 2008), or from a stochastic red noise process of equal autocorrelation to the original time series (Kang et al. 2014). The resulting surrogates are therefore representative of the assessed non-cyclical time series. Alternatively, a fourth option is applicable for EWSNet following the original authors' suggestions, where a probability larger than 0.33 (the chance that all outcomes are equally likely) is indicative of an approaching transition (Deb et al. 2022).

Resilience measures

Thus far, each of the described techniques assume that the system is at equilibrium when the stress begins to act. In ecological systems, this assumption is not likely to hold (Davidson et al. 2023) and so an alternative school of thought suggests quantifying resilience change itself rather than CSD may be more appropriate in such cases (Dakos et al. 2015). We consider resilience to be the 'capacity of system to persist and maintain its state and functions in the face of exogenous disturbance' (Hodgson et al. 2015), and consequently consists of two major components: the ability to resist and recover from disturbance (Pimm 1984). *EWSmethods* provides a fourth module containing one univariate resilience indicator (**uniJI**) and three multivariate ones (**multiJI**, **FI** and **mvi**) to estimate equilibrium free resilience change. To clarify, these methods are not EWSs but will be informative prior to a bifurcation as resilience is expected to decline as the tipping point is approached (Wissel 1984).

The functions **uniJI** and **multiJI** exploit S-map reconstruction to estimate the system's Jacobian under the empirical dynamic modelling (EDM) framework (Ushio et al. 2018, Medeiros et al. 2022, Grziwotz et al. 2023). EDM builds upon Takens theorem (Takens 1981), which suggests that if all interacting elements of a system can be simultaneously measured (all species, environmental variables, interaction and response strengths etc), then the evolution of the state of the system can be known. Unfortunately, it is unrealistic to achieve this is in empirical systems. If, however, time-delayed relationships between representative time series are estimated over a range of embedding dimensions E and lags (τ), it is possible to reconstruct a 'shadow' attractor that is topologically invariant to the true system trajectory (Sugihara et al. 2012). As the reconstructed attractor is invariant, the mathematical features of the true attractor are maintained, and one can extract information on the system's stability.

In our case, the Jacobian is interesting as tipping/bifurcation points occur when the dominant eigenvalue passes through zero from negative to positive (Strogatz 2015). It is this event that drives the phenomenon of CSD but the increase of dominant eigenvalue towards zero in general represents resilience loss. Technically, the EDM derived Jacobian is estimated around the attractor and therefore is not a direct analogue of the linear stability analysis derived Jacobians, which are informative around a bifurcation point, but does convey the advantage of estimating local Lyapunov stability. A system at equilibrium is said to be Lyapunov stable if its trajectories originating in the neighbourhood of an equilibrium point (or state) remain in the same neighbourhood (Strogatz 2015). Local Lyapunov stability is informative under both equilibrium and non-equilibrium states but does not have the same rigorous mathematical background as bifurcation theory. **uniJI** and **multiJI** calculate local Lyapunov stability (i.e. the dominant Jacobian eigenvalue) in a rolling window along the data and returns the estimate. If this estimate exceeds one, the system is unstable, while below one, it is stable. We therefore also expect that as proximity to a tipping point increases, so too will the index.

The primary difference between the univariate and multivariate form of local Lyapunov stability/Jacobian index provided by *EWSmethods* involves the target of embedding. The univariate index time embeds the focal time series against itself (Grziwotz et al. 2023) whereas the multivariate embeds across all species/time series (Ushio et al. 2018, Medeiros et al. 2022). The multivariate index is therefore sensitive to the choice of time series included in the reconstruction (Ushio et al. 2018), and the number of time series used must equal E. This limits the minimum length of uninterrupted time series required by **multiJI** at E (equal to number of measured species) + 1. The univariate index is consequently suggested to be superior if the system is not well measured (Grziwotz et al. 2023).

Multivariate alternatives to EDM-derived resilience measures exploit changing covariance between the measured time series over time. Fisher information (calculated by **FI**) estimates the amount of information data can provide on an unmeasured parameter (Fisher and Russell 1922). *EWSmethods* provides a simplified discrete time form of Fisher and Russel (1922)'s mathematic proof following Karunanithi et al. (2008):

$$\mathrm{FI} \approx 4\sum_{i=1}^{m}\left[q_i - q_{i+1}\right]^2$$

where q_i^2 is the amplitude of the probability of observing states of the system at time window i, and m is the number of possible states. States are defining by comparing the difference between temporally adjacent data windows to a reference 'uncertainty'. If the absolute difference in density is less than the reference deviation for all time series, then the windows are binned in to the same state. We recommend that this uncertainty (a.k.a 'size-of-states') is defined as the variance of each time series in a reference period or across the entire time series. Decreasing Fisher information consequently represents decreasing resilience.

And finally, the multivariate index of variability (MVI) is calculated as the square root of the dominant eigenvalue of the covariance matrix of all species by the function **mvi**. It was suggested by Brock and Carpenter (2006) as representative of resilience/stability loss as an extension of the univariate CSD indicator – variance. Therefore, akin to the variance EWS, increasing MVI represents decreasing resilience.

Example

We can illustrate the four modules of the *EWSmethods* package using one of the two datasets bundled with the package: **simsTransComms**. **simsTransComms** contains three replicate communities of five species each, simulated from a competitive Lotka–Volterra model following Dakos (2018). Each community is driven through a tipping point by increasing the carrying capacity of a low density species which mimics the appearance of an invasive species in the community. The time index of the tipping points is provided in the inflection_pt column. It is key to truncate this data set to only contain data prior to this tipping point for EWSs to have any meaningful value as a sentinel of transition (Dale and Beyeler 2001, Gsell et al. 2016). This can be achieved using the inflection_pt column of the **simTransComms$community3** community. First, we must however load the package and the **simTransComms** dataset:

```
library(EWSmethods)
data(simTransComms)
pre_simTransComms  <-  subset(simTrans
Comms$community3,time < inflection_pt)
```

This represents the data frame we will use for the remainder of this example section (Fig. 3). More detailed examples are available at: https://duncanobrien.github.io/EWSmethods/articles/ews_assessments, https://duncanobrien.github.io/EWSmethods/articles/using_ewsnet and https://duncanobrien.github.io/EWSmethods/articles/resilience_measures.

Early warning signals

To calculate univariate EWS for any one time series from this community, we would use **uniEWS**. We first need to select the EWS indicators of interest to provide to the metrics argument. Autocorrelation ('ar1') and variance (represented by the standard deviation - 'SD') are the most commonly used EWSs and have the largest body of research defining their best utility (Carpenter and Brock 2006, Dakos et al. 2012b, Patterson et al. 2021). Using these metrics, we then choose the time calculation approach (expanding), the resulting burn in period (50 data points) and the sigma threshold (2). **uniEWS** only performs assessments on univariate data but requires a two column data frame where the first column is an equally spaced time vector and the second is the time series to be assessed. We have chosen the third species here.

```
expanding_ews_eg  <-  uniEWS(data=pre_
simTransComms[,c(2,5)],
    metrics=c("ar1","SD"),
    method="expanding",
    burn_in=50,
    threshold=2)
  plot(expanding_ews_eg,
  y_lab="Density")
```

The resulting *ggplot* (Wickham 2016) (Fig. 4) called by **plot** shows that warnings are generated from timepoint 171 onwards for all EWSs following multiple consecutive 'signals'.

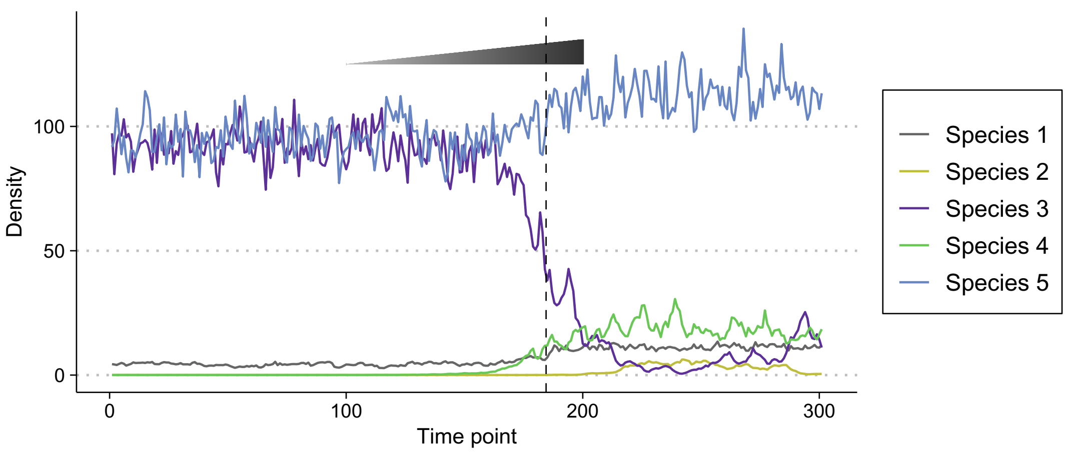

Figure 3. The simulated simTransComms$community3 community plotted against time. Species 4's carrying capacity is gradually increased within the time interval 100–200 (as represented by the expanding wedge) to mimic the appearance of an invasive species. This drives a community transition with the inflection point indicated by a vertical dashed line.

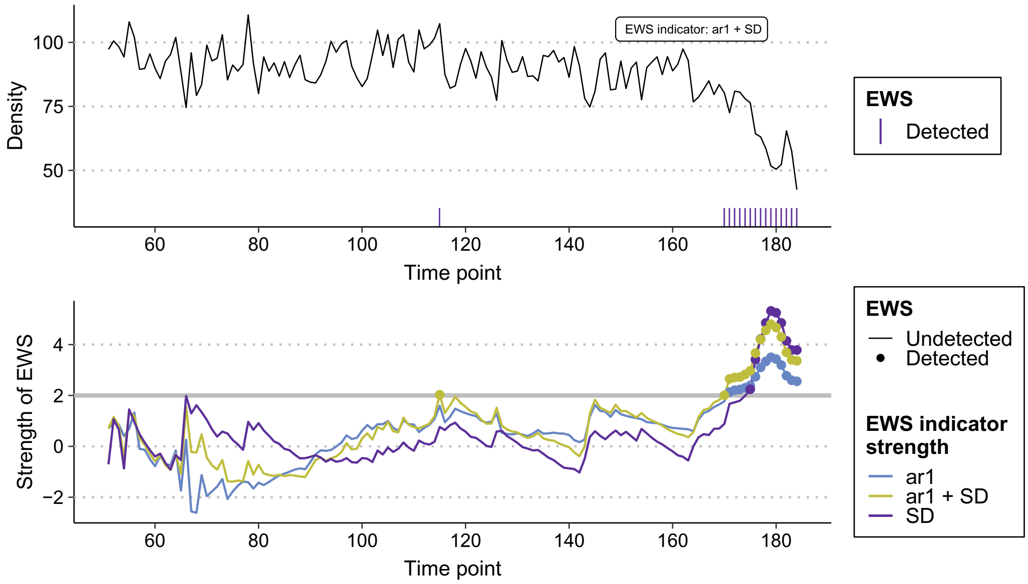

Figure 4. Expanding window assessment of species three in the pre_simTransComms dataset using the univariate autocorrelation and variance early warning signal indicators. The figure is a direct output of the *EWSmethods* **plot** S3 method on an **uniEWS** generated object. The top panel depicts the raw time series and the presence of a signal from the annotated indicator. The lower panel visualises the strength of each indicator through time and the threshold level. A signal is indicated when the indicator strength exceeds this threshold value.

The single signal for the 'ar1 + SD' indicator at timepoint 115 is not sufficient to be a warning.

To expand the assessment to include information from all time series, we require the use of **multiEWS**. The single difference in the function's parameterisation is that the input data frame must contain more than two columns (one time sequence column and two or more time series). By default, all indicators are returned.

To compute multivariate EWSs using the rolling method, the function would be written as thus, specifying the method and winsize as a percentage of the time series' length:

```
multi_ews_eg_roll <- multiEWS(data=pre_
simTransComms[,2:7],
    method="rolling",
    winsize=50)
plot(multi_ews_eg_roll)
```

All indicators are positively correlated with time (excluding mafSD) but the strength of correlation varies (mean tau = 0.55, Fig. 5). However, each indicator does increase prior to transition even if this is not universally represented in the tau coefficients.

Rolling window EWSs are hampered in that the strength of Kendall tau is ultimately arbitrary without a reference, and the overlapping windows renders typical significance tests inappropriate (Dakos et al. 2012a). However, using pseudo-random surrogate time series representative of the focal time series, and estimation of the relatively strength of correlation can be calculated. To achieve this in EWSmethods, we can use **perm_rollEWS**, specifying the method of permutation to be a red noise process, the number of permutations to be 500 and the approach to be multivariate:

```
perm_ews_eg_roll          <-          perm_
rollEWS(data=pre_simTransComms[,2:7],
   metrics=c("meanAR","maxAR","meanSD",
     "maxSD","eigenMAF","mafAR",
     "mafSD","pcaAR","pcaSD",
     "eigenCOV","maxCOV","mutINFO"),
     variate="multi",
     perm.meth="red.noise",
     winsize=50,
     iter=500)
  print(perm_ews_eg_roll$EWS$cor)
    meanAR  maxAR   meanSD  maxSD eigen-
MAF mafAR
  tau      0.8705002  0.6423562  0.769051
0.4824684 0.7812062 0.6517064
  perm_pvalue       0.0000000   0.0060000
0.000000 0.1620000 0.0440000 0.0120000
     mafSD pcaAR    pcaSD  eigenCOV max-
COV mutINFO
  tau     -0.1584853 0.6998597 0.5357644
0.5357644 0.658719 0.2130467
  perm_pvalue        0.4580000  0.0040000
0.0020000 0.0020000 0.000000 0.0820000
```

Using surrogate data, we can see that most indicators are in the top 95% of surrogate correlation strengths (p-values less than 0.05) and so we can reject the null that these correlations are spurious.

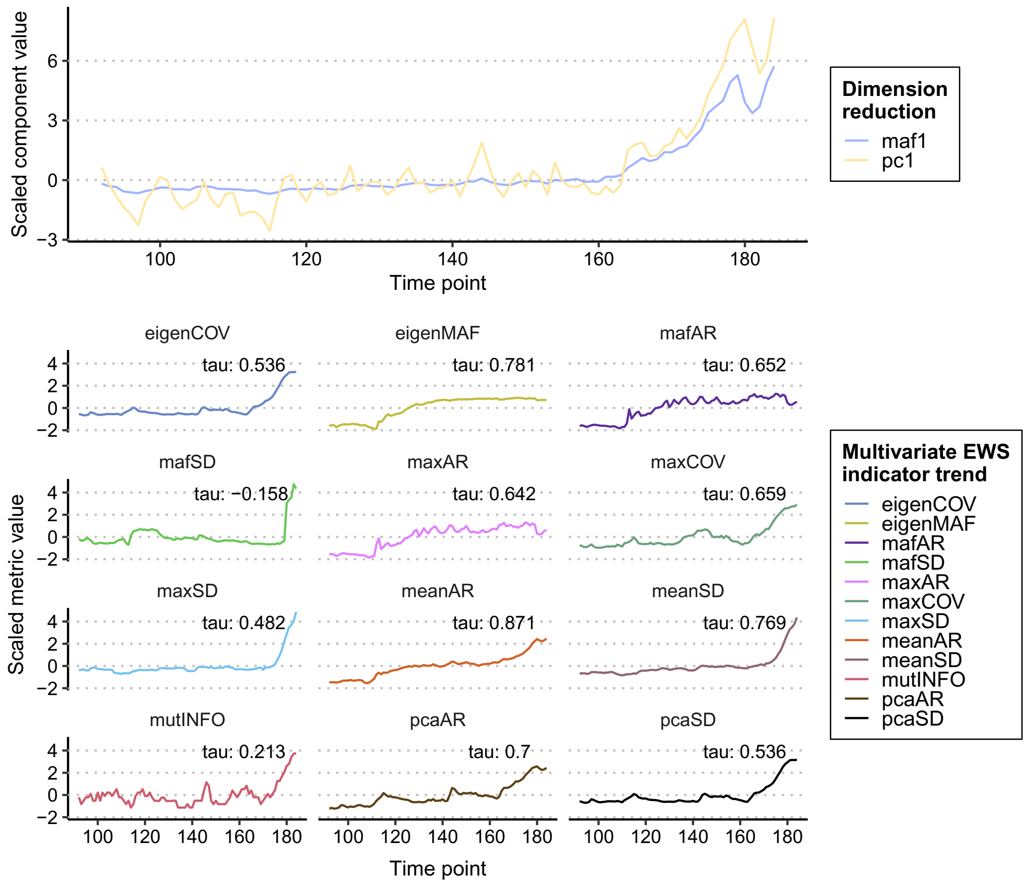

Figure 5. Rolling window assessment of the entire pre_simTransComms community using multivariate early warning signal indicators. The figure is a direct output of the *EWSmethods* **plot** S3 method on a **multiEWS** generated object. The top panel plots the raw dimension reductions from which certain indicators are estimated. The lower panel visualises the trend in each indicator through time and reports the Kendall's tau correlation coefficient.

To perform the assessment process using expanding windows, **multiEWS** simply requires a change of method argument and the provision of burn in and threshold.

```
multi_ews_eg_expand                          <-
multiEWS(data=pre_simTransComms[,2:7],
    method="expanding",
    burn_in=50,
    threshold=2)
plot(multi_ews_eg_expand)
```

Warnings are generated throughout this assessment with two consistently signalled periods at timepoints 110 and 175 (Fig. 6). This highlights the usefulness of expanding windows over rolling as the exact time point of warning can be determined, but supports Weinans et al.'s (2021) suggestion that there is no superior multivariate EWS indicator; the best fit depends on the scenario the system is subject to.

EWSNet

EWSNet requires initialisation using **ewsnet_init** due to its Python backend. At the start of each R (www.r-project.org) session, **ewsnet_init** must be called and a consistent envname provided. When the function is run for the first time on a new machine, Python will be downloaded alongside the critical Python packages and a new environment (envname) created. The user will be prompted to agree to this by default (when the auto argument is FALSE) to ensure the files will not be accidentally downloaded if undesired. For future sessions, providing the same envname will result in the original environment being activated rather than redownloading all files.

```
ewsnet_init(envname="EWSNET_env", pip_
ignore_installed=FALSE, auto=FALSE)
```

The large file size of the model weights (~220mb) also means that *EWSmethods* does not come bundled with them. The user is required to call the **ewsnet_reset** function which will prompt confirmation that the weights are to be downloaded from https://ewsnet.github.io.

```
ewsnet_reset(remove_weights=FALSE)
```

Once initiated, **ewsnet_predict** will accept a vector timeseries (note no time sequence is required) alongside the model

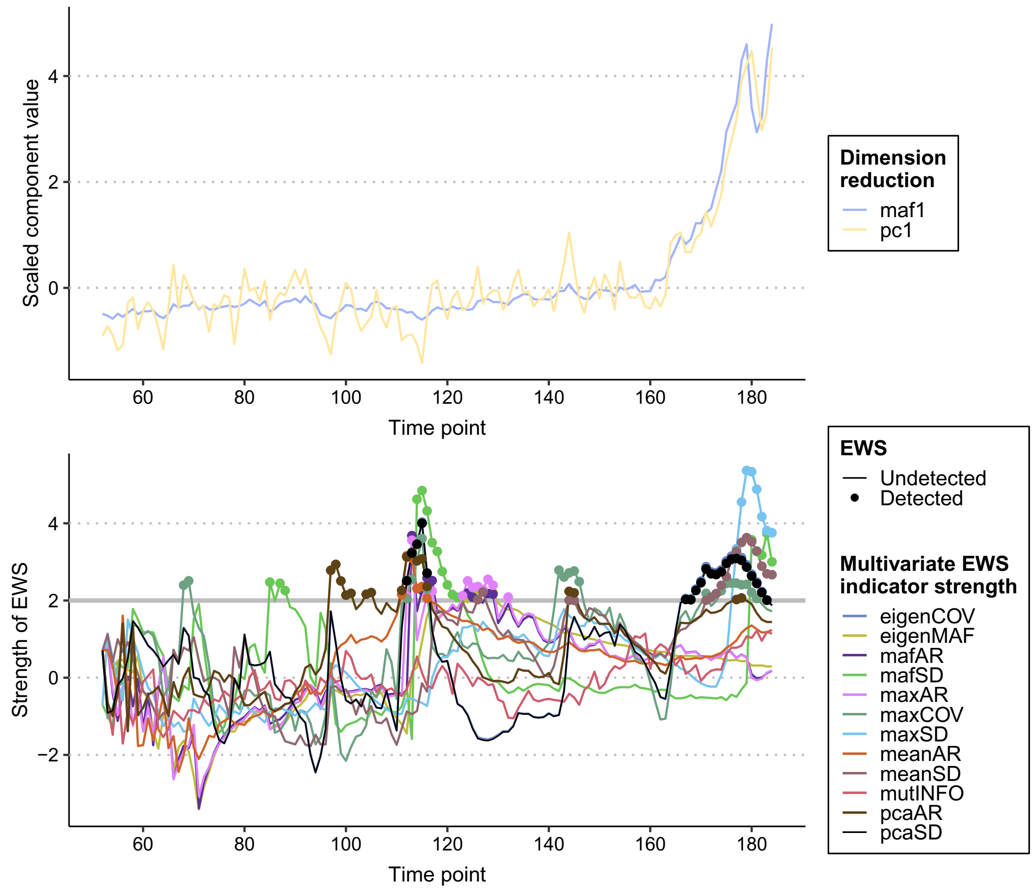

Figure 6. Expanding window assessment of the entire pre_simTransComms community using multivariate early warning signal indicators. The figure is a direct output of the *EWSmethods* **plot** S3 method on a **multiEWS** generated object.

weights to use. These model weights are subset based on scaling (scaled vs unscaled) and the number of models to average over (ensemble). We recommend using scaled weights averaged over the maximum ensemble size (25) for most robust predictions.

```
ewsnet_prediction <- ewsnet_predict(x=pre_simTransComms[,5], scaling=TRUE, ensemble=25, envname="EWSNET_env")
  print(ewsnet_prediction)
  pred     no_trans_prob  smooth_trans_prob  crt_trans_prob
  Critical    Transition          0.196918
0.1813867    0.6216951
```

A critical transition has subsequently been predicted with a 62% probability indicating that a sudden tipping point is imminent.

Resilience measures

Each resilience measure is returned by a unique function but share the similar parameterisation as **uniEWS** and **multiEWS**. Slight differences are available for **uniJI**, **multiJI** and **FI** regarding the time delay embedding and tightening level respectively. **FI** also requires an additional argument specifying the vector of size-of-states. We can estimate each measure for our community as so.

```
multi_JI_eg <- multiJI(data=pre_simTransComms[,2:7], winsize=50, scale=TRUE)
  uni_JI_eg <- uniJI(data=pre_simTransComms[,c(2,5)], winsize=50, E=1, scale=TRUE)
  mvi_eg <- mvi(data=pre_simTransComms[,2:7], winsize=50)
  sost_eg <- t(apply(pre_simTransComms[,3:7], MARGIN=2, FUN=sd))
  #transpose required to ensure a 1 x n matrix of size-of-states to be taken by `FI`
  fi_eg <- FI(data=pre_simTransComms[,2:7], sost=sost_eg, winsize=50, TL=90)$FI
```

Figure 7 resultingly shows the trend of each resilience measure prior to the tipping point and relative to the increasing stress. All measures other than Fisher information increase

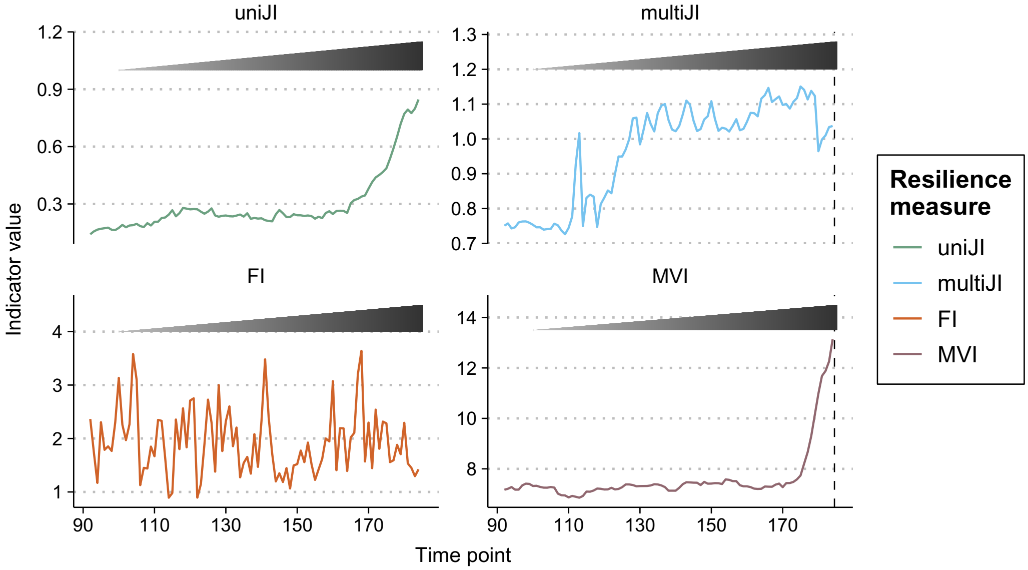

Figure 7. Resilience measure assessment of the pre_simTransComms community using multivariate indicators, and a univariate assessment of species three. The figure is not a direct output of the *EWSmethods* functions but is built using the data returned by **uniJI**, **multiJI**, **FI** and **mvi**. Vertical dotted lines are the estimated inflection point and the expanding wedge represents the increasing stress.

prior to the tipping point with the multivariate Jacobian index displaying trends earliest. Similarly, the univariate Jacobian index also reaches 1.0 at the tipping point, matching its conceptual behaviour in Grziwotz et al. (2023). Fisher information does not, however, qualitatively decrease prior to the tipping point though this may stem from an inappropriate window size or size-of-state.

Conclusion

The ability to use accessible and easy to interpret tools are key for ecological monitoring. In this note we present *EWSmethods*, an R package (www.r-project.org) consolidating the simplest methods of EWS assessments into a coherent suite of metrics and visualisations. Each function is consistent in its parameterisations, terminology, and output to allow any user to interpret the assessment confidently, regardless of the data dimensionality or EWS approach.

It would however be remiss to overlook the pivotal *earlywarnings* package and work of Dakos et al. (2012a). *EWSmethods* innovates on *earlywarnings* by providing alternative calculations (rolling vs expanding windows) and data types (univariate vs multivariate), but does not provide the additional modelling techniques *earlywarnings* supports (diffusion-drift-jump models, BDS tests etc). We direct readers to that package on CRAN (https://cran.r-project.org/web/packages/earlywarnings) for the typical univariate rolling window EWS approach due to the additional modelling capabilities it provides. *EWSmethods* better supports multivariate analyses and standardises across univariate EWSs, multivariate EWSs and machine learning models to allow comparability. It also provides access to purpose-built machine learning models not otherwise available to R (www.r-project.org) users. Consequently, users are able to explore an ensemble of generic forecasting methods to identify oncoming transitions and tipping points in their system. Alternatively, if the reader is more interested in explicitly forecasting a time series future, then we suggest the EWS package (https://cran.r-project.org/web/packages/EWS) for a frequentist approach, or the work of Laitinen et al. (2021) for a probabilistic, as an alternative framework to those presented here.

Generic approaches also facilitate wider research interest into the universal challenge of identifying oncoming tipping points. Resilience-based approaches are critical for the management of globally imperilled systems (Folke et al. 2010, Oliver et al. 2015, Capdevila et al. 2022) but are applicable in other disciplines. Remotely sensed data could allow global level tipping point assessments for example (Forzieri et al. 2022), individual mortality risk may be detectable (Cailleret et al. 2019) or positive thresholds can be encouraged (Lenton et al. 2022). The low barrier to entry that *EWSmethods* provides for R (www.r-project.org) users can aid the development of these developing research avenues.

To cite *EWSmethods* or acknowledge its use, cite this Software note as follows, substituting the version of the application that you used for 'ver. 1.1.2':

O'Brien D. A. et al. 2023. *EWSmethods*: an R package to forecast tipping points at the community level using early warning signals, resilience measures, and machine learning models. – Ecography 2023: e06674 (ver. 1.1.2).

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Author contributions

Duncan A. O'Brien: Conceptualization (equal); Funding acquisition (equal); Methodology (equal); Software (equal); Visualization (equal); Writing – original draft (lead). **Smita Deb**: Conceptualization (equal); Methodology (equal); Software (equal); Validation (equal); Writing – original draft (supporting); Writing – review and editing (equal). **Sahil Sidheekh**: Methodology (equal); Software (equal); Writing – review and editing (equal). **Narayanan C. Krishnan**: Methodology (equal); Software (equal); Supervision (equal); Writing – review and editing (equal). **Partha Sharathi Dutta**: Methodology (equal); Supervision (equal); Writing – review and editing (equal). **Christopher F. Clements**: Conceptualization (equal); Funding acquisition, Methodology (equal); Supervision (equal); Writing – review and editing (equal).

Transparent peer review

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Data availability statement

No data was used in this manuscript other than those provided in the EWSmethods R package (https://CRAN.R-project.org/package=EWSmethods).

References

Baruah, G., Clements, C. F. and Ozgul, A. 2020. Eco-evolutionary processes underlying early warning signals of population declines. – J. Anim. Ecol. 89: 436–448.

Baruah, G., Ozgul, A. and Clements, C. F. 2022. Community structure determines the predictability of population collapse. – J. Anim. Ecol. 91: 1880–1891.

Besson, M., Alison, J., Bjerge, K., Gorochowski, T. E., Høye, T. T., Jucker, T., Mann, H. M. R. and Clements, C. F. 2022. Towards the fully automated monitoring of ecological communities. – Ecol. Lett. 25: 2753–2775.

Biggs, R., Carpenter, S. R. and Brock, W. A. 2009. Turning back from the brink: Detecting an impending regime shift in time to avert it. – Proc. Natl Acad. Sci. USA 106: 826–831.

Bradley, E. and Kantz, H. 2015. Nonlinear time-series analysis revisited. – Chaos 25: 097610.

Brock, W. and Carpenter, S. R. 2006. Variance as a leading indicator of regime shift in ecosystem services. – Ecol. Soc. 11: 9.

Bury, T. M., Sujith, R. I., Pavithran, I., Scheffer, M., Lenton, T. M., Anand, M. and Bauch, C. T. 2021. Deep learning for early warning signals of tipping points. – Proc. Natl Acad. Sci. USA 118: e2106140118.

Cailleret, M. et al. 2019. Early-warning signals of individual tree mortality based on annual radial growth.

Capdevila, P., Noviello, N., McRae, L., Freeman, R. and Clements, C. F. 2022. Global patterns of resilience decline in vertebrate populations. – Ecol. Lett. 25: 240–251.

Carpenter, S. R. and Brock, W. A. 2006. Rising variance: a leading indicator of ecological transition. – Ecol. Lett. 9: 311–318.

Carpenter, S. R., Cole, J. J., Pace, M. L., Batt, R., Brock, W. A., Cline, T., Coloso, J., Hodgson, J. R., Kitchell, J. F., Seekell, D. A., Smith, L. and Weidel, B. 2011. Early warnings of regime shifts: a whole-ecosystem experiment. – Science 332: 1079.

Chen, S., O'Dea, E. B., Drake, J. M. and Epureanu, B. I. 2019. Eigenvalues of the covariance matrix as early warning signals for critical transitions in ecological systems. – Sci. Rep. 9: 2572.

Cheng, Z., Liu, F., Zhang, X.-P. and Wang, W. 2008. Robustness analysis of cellular memory in an autoactivating positive feedback system. – FEBS Lett. 582: 3776–3782.

Clements, C. F. and Ozgul, A. 2016. Including trait-based early warning signals helps predict population collapse. – Nat. Commun. 7: 10984.

Clements, C. F. and Ozgul, A. 2018. Indicators of transitions in biological systems. – Ecol. Lett. 21: 905–919.

Clements, C. F., Blanchard, J. L., Nash, K. L., Hindell, M. A. and Ozgul, A. 2017. Body size shifts and early warning signals precede the historic collapse of whale stocks. – Nat. Ecol. Evol. 1: 188.

Clements, C. F., McCarthy, M. A. and Blanchard, J. L. 2019. Early warning signals of recovery in complex systems. – Nat. Commun. 10: 1681.

Dakos, V., Scheffer, M., Van Nes, E. H., Brovkin, V., Petoukhov, V. and Held, H. 2008. Slowing down as an Early Warning Signal for abrupt climate change. – Proc. Natl Acad. Sci. USA 105: 14308–14312.

Dakos, V., van Nes, E. H., Donangelo, R., Fort, H. and Scheffer, M. 2010. Spatial correlation as leading indicator of catastrophic shifts. – Theor. Ecol. 3: 163–174.

Dakos, V., Carpenter, S. R., Brock, W. A., Ellison, A. M., Guttal, V., Ives, A. R., Kéfi, S., Livina, V., Seekell, D. A., van Nes, E. H. and Scheffer, M. 2012a. Methods for detecting early warnings of critical transitions in time series illustrated using simulated ecological data. – PLoS One 7: e41010.

Dakos, V., van Nes, E. H., D'Odorico, P. and Scheffer, M. 2012b. Robustness of variance and autocorrelation as indicators of critical slowing down. – Ecology 93: 264–271.

Dakos, V., Carpenter, S. R., van Nes, E. H. and Scheffer, M. 2015. Resilience indicators: prospects and limitations for early warnings of regime shifts. – Philos. Trans. R. Soc. B 370: 20130263.

Dakos, V. 2018. Identifying best-indicator species for abrupt transitions in multispecies communities. – Ecol. Indic. 94: 494–502.

Dale, V. H. and Beyeler, S. C. 2001. Challenges in the development and use of ecological indicators. – Ecol. Indic. 1: 3–10.

Davidson, T. A., Sayer, C. D., Jeppesen, E., Søndergaard, M., Lauridsen, T. L., Johansson, L. S., Baker, A. and Graeber, D. 2023. Bimodality and alternative equilibria do not help explain long-term patterns in shallow lake chlorophyll-a. – Nat. Commun. 14: 398.

De Gooijer, J. G. and Hyndman, R. J. 2006. 25 years of time series forecasting. – Int. J. Forecast. 22: 443–473.

Deb, S., Sidheekh, S., Clements, C. F., Krishnan, N. C. and Dutta, P. S. 2022. Machine learning methods trained on simple models can predict critical transitions in complex natural systems. – R. Soc. Open Sci. 9: 211475.

Drake, J. M. and Griffen, B. D. 2010. Early warning signals of extinction in deteriorating environments. – Nature 467: 456–459.

Drake, J. M., Brett, T. S., Chen, S., Epureanu, B. I., Ferrari, M. J., Marty, É., Miller, P. B., O'Dea, E. B., O'Regan, S. M., Park, A. W. and Rohani, P. 2019. The statistics of epidemic transitions. – PLoS Comput. Biol. 15: e1006917.

Enni, S. A. and Herrie, M. B. 2021. Turning biases into hypotheses through method: a logic of scientific discovery for machine learning. – Big Data Soc. 8: 20539517211020776.

Fisher, R. A. and Russell, E. J. 1922. On the mathematical foundations of theoretical statistics. – Philos. Trans. R. Soc. A 222: 309–368.

Folke, C., Carpenter, S. R., Walker, B., Scheffer, M., Chapin, T. and Rockström, J. 2010. Resilience thinking. – Ecol. Soc. 15.

Forzieri, G., Dakos, V., McDowell, N. G., Ramdane, A. and Cescatti, A. 2022. Emerging signals of declining forest resilience under climate change. – Nature 608: 534–539.

Fraedrich, K. 1978. Structural and stochastic analysis of a zero-dimensional climate system. – Q. J. R. Meteorol. Soc. 104: 461–474.

Guttal, V. and Jayaprakash, C. 2008. Changing skewness: an early warning signal of regime shifts in ecosystems. – Ecol. Lett. 11: 450–460.

Grziwotz, F., Chang, C.-W., Dakos, V., van Nes, E. H., Schwarzländer, M., Kamps, O., Heßler, M., Tokuda, I. T., Telschow, A. and Hsieh, C. 2023. Anticipating the occurrence and type of critical transitions. – Sci. Adv. 9: eabq4558.

Gsell, A. S., Scharfenberger, U., Özkundakci, D., Walters, A., Hansson, L.-A., Janssen, A. B. G., Nõges, P., Reid, P. C., Schindler, D. E., Van Donk, E., Dakos, V. and Adrian, R. 2016. Evaluating early-warning indicators of critical transitions in natural aquatic ecosystems. – Proc. Natl Acad. Sci. USA 113: E8089–E8095.

Harris, M. J., Hay, S. I. and Drake, J. M. 2020. Early warning signals of malaria resurgence in Kericho, Kenya. – Biol. Lett. 16: 20190713.

Held, H. and Kleinen, T. 2004. Detection of climate system bifurcations by degenerate fingerprinting. – Geophys. Res. Lett. 113.

Hodgson, D., McDonald, J. L. and Hosken, D. J. 2015. What do you mean, 'resilient'? – Trends Ecol. Evol. 30: 503–506.

Humphries, G., Magness, D. and Huettmann, F. 2018. Machine learning for ecology and sustainable natural resource management.

Jucker, T. et al. 2017. Allometric equations for integrating remote sensing imagery into forest monitoring programmes. – Global Change Biol. 23: 177–190.

Kang, Y., Belušić, D. and Smith-Miles, K. 2014. Detecting and classifying events in noisy time series. – J. Atmos. Sci. 71: 1090–1104.

Karunanithi, A. T., Cabezas, H., Frieden, B. R. and Pawlowski, C. W. 2008. Detection and assessment of ecosystem regime shifts from fisher information. – Ecol. Soc.. 71.

Kéfi, S., Dakos, V., Scheffer, M., Van Nes, E. H. and Rietkerk, M. 2013. Early warning signals also precede non-catastrophic transitions. – Oikos 122: 641–648.

Kleinen, T., Held, H. and Petschel-Held, G. 2003. The potential role of spectral properties in detecting thresholds in the Earth system: application to the thermohaline circulation. – Ocean Dyn. 53: 53–63.

Kuehn, C. 2011. A mathematical framework for critical transitions: bifurcations, fast–slow systems and stochastic dynamics. – Phys. D: Nonlinear Phenom. 240: 1020–1035.

Laitinen, V., Dakos, V. and Lahti, L. 2021. Probabilistic early warning signals. – Ecol. Evol.11: 14101–14114.

Lenton, T. M., Benson, S., Smith, T., Ewer, T., Lanel, V., Petykowski, E., Powell, T. W. R., Abrams, J. F., Blomsma, F. and Sharpe, S. 2022. Operationalising positive tipping points towards global sustainability. – Global Sustain. 5: e1.

Lever, J. J., van de Leemput, I. A., Weinans, E., Quax, R., Dakos, V., van Nes, E. H., Bascompte, J. and Scheffer, M. 2020. Foreseeing the future of mutualistic communities beyond collapse. – Ecol. Lett. 23: 2–15.

Ludwig, D., Jones, D. D. and Holling, C. S. 1978. Qualitative analysis of insect outbreak systems: the spruce budworm and forest. – J. Anim. Ecol. 47: 315–332.

MathWorks, I. 2022. MATLAB. – https://www.mathworks.com

McSharry, P. E., Smith, L. A. and Tarassenko, L. 2003. Prediction of epileptic seizures: are nonlinear methods relevant? – Nat. Med. 9: 241–242.

Medeiros, L. P., Allesina, S., Dakos, V., Sugihara, G. and Saavedra, S. 2022. Ranking species based on sensitivity to perturbations under non-equilibrium community dynamics. – Ecol. Lett. 26: 170–183.

Meinson, P., Idrizaj, A., Nõges, P., Nõges, T. and Laas, A. 2015. Continuous and high-frequency measurements in limnology: history, applications, and future challenges. – Environ. Rev. 24: 52–62.

O'Brien, D. A. and Clements, C. F. 2022. Early warning signal reliability varies with COVID-19 waves. – Biol. Lett. 17: 20210487.

O'Brien, D. A., Deb, S., Gal, G., Thackeray, S. J., Dutta, Matsuzaki, S. S., May, L. and Clements, C. F. 2023. Early warning signals are hampered by a lack of critical transitions in empirical lake data. – bioRxiv 2023.05.11.540304.

Oliver, T. H., Heard, M. S., Isaac, N. J. B., Roy, D. B., Procter, D., Eigenbrod, F., Freckleton, R., Hector, A., Orme, C. D. L., Petchey, O. L., Proença, V., Raffaelli, D., Suttle, K. B., Mace, G. M., Martín-López, B., Woodcock, B. A. and Bullock, J. M. 2015. Biodiversity and resilience of ecosystem functions. – Trends Ecol. Evol. 30: 673–684.

Pace, M. L., Carpenter, S. R. and Cole, J. J. 2015. With and without warning: managing ecosystems in a changing world. – Front. Ecol. Environ. 13: 460–467.

Patterson, A. C., Strang, A. G. and Abbott, K. C. 2021. When and where we can expect to see early warning signals in multispecies systems approaching tipping points: insights from theory. – Am. Nat. 198: E12–E26.

Pimm, S. L. 1984. The complexity and stability of ecosystems. – Nature 307: 321–326.

Quax, R., Kandhai, D. and Sloot, P. M. A. 2013. Information dissipation as an early-warning signal for the Lehman Brothers collapse in financial time series. – Sci. Rep. 3: 1898.

Rogers, B. M., Solvik, K., Hogg, E. H., Ju, J., Masek, J. G., Michaelian, M., Berner, L. T. and Goetz, S. J. 2018. Detecting early warning signals of tree mortality in boreal North America using multiscale satellite data. – Global Change Biol. 24: 2284–2304.

Scheffer, M., Bascompte, J., Brock, W. A., Brovkin, V., Carpenter, S. R., Dakos, V., Held, H., van Nes, E. H., Rietkerk, M. and Sugihara, G. 2009. Early-warning signals for critical transitions. – Nature 461: 53–59.

Scheffer, M., Carpenter, S. R., Lenton, T. M., Bascompte, J., Brock, W., Dakos, V., van de Koppel, J., van de Leemput, I. A., Levin, S. A., van Nes, E. H., Pascual, M. and Vandermeer, J. 2012. Anticipating critical transitions. – Science 338: 344–348.

Schreuder, M. J., Hartman, C. A., George, S. V., Menne-Lothmann, C., Decoster, J., van Winkel, R., Delespaul, P., De Hert, M.,

Derom, C., Thiery, E., Rutten, B. P. F., Jacobs, N., van Os, J., Wigman, J. T. W. and Wichers, M. 2020. Early warning signals in psychopathology: what do they tell? – BMC Med. 18: 269.
Southall, E., Tildesley, M. J. and Dyson, L. 2022. How early can an upcoming critical transition be detected? – medRxiv 2022.05.27.22275693.
Strogatz, S. H. 2015. Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering. – CRC Press.
Sugihara, G., May, R., Ye, H., Hsieh, C., Deyle, E., Fogarty, M. and Munch, S. 2012. Detecting causality in complex ecosystems. – Science 338: 496–500.
Suweis, S. and D'Odorico, P. 2014. Early warning signs in social-ecological networks. – PLoS One 9: e101851.
Takens, F. 1981. In: Rand, D. and Young, L.-S. (eds), Detecting strange attractors in turbulence. – Dynamical Systems and Turbulence, pp. 366–381.
Theiler, J., Eubank, S., Longtin, A., Galdrikian, B. and Doyne Farmer, J. 1992. Testing for nonlinearity in time series: the method of surrogate data. – Phys. D: Nonlinear Phenom. 58: 77–94.
Ushey, K., Allaire, J. J. and Tang, Y. 2022. reticulate: interface to "Python". – https://rstudio.github.io/reticulate/
Ushio, M., Hsieh, C., Masuda, R., Deyle, E. R., Ye, H., Chang, C.-W., Sugihara, G. and Kondoh, M. 2018. Fluctuating interaction network and time-varying stability of a natural fish community. – Nature 554: 360–363.
Weinans, E., Lever, J. J., Bathiany, S., Quax, R., Bascompte, J., van Nes, E. H., Scheffer, M. and van de Leemput, I. A. 2019. Finding the direction of lowest resilience in multivariate complex systems. – J. R. Soc. Interf. 16: 20190629.
Weinans, E., Quax, R., van Nes, E. H. and van de Leemput, I. A. 2021. Evaluating the performance of multivariate indicators of resilience loss. – Sci. Rep. 11: 9148.
Wickham, H. 2016. ggplot2: elegant graphics for data analysis. – Springer-Verlag New York.
Wissel, C. 1984. A universal law of the characteristic return time near thresholds. – Oecologia 65: 101–107.